Algebra Mastery Through Relevant Applications in Grades 6-10

International Center for Leadership in Education

Acknowledgments

The International Center for Leadership in Education wishes to thank the following authors for their contributions to this kit:

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Overview

Proficiency in high school mathematics can be used as a gauge for future success in education and employment. Moreover, research indicates that students who are successful in algebra are more likely to take higher level math courses in high school and college. As a result of these findings, there is a relatively new pressure in mathematics education – to get students through algebra.

Reform efforts to improve math performance typically focus on:

- "fixing" first-year algebra in high school
- moving the start of algebra instruction to the middle grades
- adopting a new textbook series

While these efforts may be helpful, they do not address instruction. The purpose of this resource kit is not to add one more thing *to do* in math class but to provide things teachers *can do* to help students think independently in math class. The kit contains research on teaching and learning algebra and specific strategies to teach algebra through relevant applications.

One tool included for improving the rigor and relevance of instruction is the International Center for Leadership in Education's Rigor/ Relevance Framework. Teaching algebra at a mastery level through relevancy is achieved by using lessons that are interdisciplinary or that include real-world situations. Instruction that is designed and delivered with this Framework in mind results in an increase in student engagement and academic achievement.

Understanding and using algebra are dependent on knowing a number of fundamental concepts, such as patterns, equality, and variable. This resource kit contains research on why students struggle to understand these concepts and suggestions on how to help them.

A New Approach

The teach-test classroom, in which students solve the equation that the teacher presents standing in front of the classroom, is a thing of the past. Employers and colleges want problem solvers who can see the big picture, work collaboratively to find a solution, and explain that solution to others. This resource kit is a springboard for evolving mathematics instruction. It addresses different learning styles, offers strategies that are specific to the math classroom, and includes *actual lessons* that have been used by math teachers. It is a resource that both beginning and veteran teachers will use many times — adding their own thoughts and experiences to what the kit offers to suit their teaching style and their students.

Chapter 1: Algebra for a New Generation presents the reality of the 21st century classroom. Algebra is a priority for K-12 education.

Chapter 2: Rigor/Relevance Framework™ provides an overview of the Framework and examples of how it applies to the algebra curriculum. The chapter gives examples of typical lessons and shows how they can be adapted to Quadrant D.

Chapter 3: Struggling Students delves into the research around learning styles and how students learn algebra. This research supports the need to make learning algebra more relevant.

Chapter 4: Strategies for Teaching and Learning Algebra offers 42 strategies that can be used in math class. The strategies, developed and implemented by math teachers, are categorized by literacy, learning styles, student engagement, instruction, and administrivia.

Chapter 5: Mathematical Inquiry describes this instructional strategy in detail. This approach encourages setting up experiences

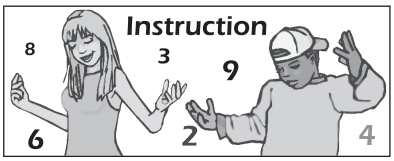
in math class that allow students to test hypotheses and communicate mathematically.

Chapter 6: Professional Development Activities has tools for teachers and administrators to evaluate current programs and plan the future direction of the school's math program. There are 20 activities and five best practices that can be implemented to raise awareness of the need to hold high expectations for all students.

Chapter 7: Gold Seal Lessons has 36 Gold Seal Lessons that were developed based on the Rigor/Relevance Framework. The majority are high rigor/high relevance (Quadrant D). These lessons, created and implemented by math teachers, can be replicated and/or adapted for classroom use.

There are two **Appendices** in the resource kit. The first one describes the Quantile Framework® for Mathematics, a tool for measuring mathematical ability. The second one explains the I CAN Learn® Education Program, an interactive, computer-based experience that individualizes instruction. Each has specific tools and measures for algebra.

Also included in the kit are two DVDs and a CD. The CD contains Chapters 4 and 6, the full text of all Gold Seal Lessons in Chapter 7, and several other items. DVD #1 has a 50-minute presentation by Dr. Willard R. Daggett entitled *Global Competition in Math and Science*. DVD #2 has a 35-minute presentation by Dr. Richard D. Jones, *Introduction to the Rigor/Relevance Framework* and the 11-minute math inquiry, *How Much Does a Triangle Cost These Days?*, presented by Dr. Dan Hurwitz.



Strategy: Vary the Degree of Difficulty

Topic: Solving Equations/Factoring

Defining the Strategy

The goal of this activity is to put the responsibilityinto the hands of students for creating methods for solving problems on their own. Provide students with equations that vary in the degree of difficulty (or number of steps, for example). This not only provides students with lower level questions (on the knowledge taxonomy) to help them review previously learned concepts, but also challenges learners to stretch themselves and tackle more difficult questions in a non-threatening way.

Teaching the Strategy

Objectives

- Students will solve 4 or 5 equations that vary from one step to multi-step.
- Students will solve for x mentally, and then describe a method for showing their work algebraically.

Materials

A variety of equations that vary in degree of difficulty

Preparation

Select four or five equations that range from a one-step equation to a three or four step equation.

Write these equations on the board or on a handout.

Assign groups of students or individuals to solve all problems using any method they choose.

Ask students to share their methods of solving the equations on the board or in small groups.

Once strategies are shared, you can describe your method for solving multi-step equations.

Example

Typically, when teaching students to solve equation, a one-step equation may be given:

$$X + 8 = 20$$

Many students will be able to solve this equation mentally, however math teachers know that they will also need to show their work and will thus ask them to write them down!

$$\begin{array}{r} X + 8 = 20 \\ \underline{\quad - 8 = - 8} \\ X = 12 \end{array}$$

And then to check their work:

$$\begin{array}{r} 12 + 8 = 20 \\ 20 = 20 \text{ check!} \end{array}$$

Students will then be given 20 questions that ask them to practice their new skills, with different numbers accordingly. A more general approach to having students solve equations is to present them with a set of equations that vary in degree of difficultly. For example:

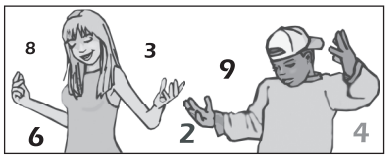
$X + 8 = 20$	$2x - 18 = -30$	$5x + 10 = 6x - 32$	$2x + 16 = 4x + 6$

The challenge for the students is to solve each of these equations using their own method. Students will probably move through the first two equations quickly and may struggle a little more with the last two examples. In many cases, we supply the method we think works best, but this may not be developmentally appropriate for the student, nor match up with their learning style. This method can be applied to factoring questions as well.

$7x + 28$	$5x^3 - 35x^2 + 10x$	$16x^2y + 8x^4y^2 - 12x^4y^5$	$x^2 + 2x - 24$

Please see the CD for the two Student Handouts in landscape.

The advantage of this method is getting the students to work together to come up with their own methods. They may solve the equations mentally, but they should be encouraged to articulate their answers in written and verbal form. Students will begin to look at the problem as a whole and apply the concepts. Too often, students apply the rules that were dictated by the teacher and the purpose of the procedure is lost.



Student Handout: *Solving Equations*

Solve:

X + 8 = 20	2 x – 18 = - 30	5 x + 10 = 6 x – 32	2 x + 16 = 4 x + 6

The Structure of an Inquiry

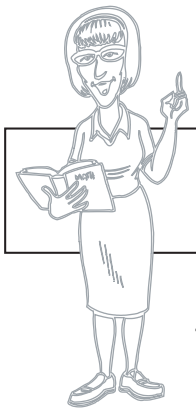
Question –	Facilitator states the topic.
Exploration –	The class separates into groups. The groups are given the necessary materials. They explore.
Preparation –	Explorations are drawn to conclusions. Presentations are prepared.
Presentation –	Each group shows its work and results in the Math Congress.
Follow-up –	There is a general class discussion with possible extensions and assessment.

Why Use Mathematical Inquiries?

Most educators can easily imagine the following tableau: during mathematics instruction, one student answers a question posed by the teacher. The teacher then asks, "How did you get that?" A look of panic covers the student's face, since she or he is certain a mistake must have been the cause for this question.

Most students (and some teachers) believe that in mathematics an answer is either right or wrong, and that the essential educational content of an answer is, therefore, its correctness. So a follow-up question like the teacher's above can only be a sign of a wrong response.

Misconceptions like this about mathematics education have been summarized in what T. Christine Stevens called "The Implicit Curriculum." Here are three components of that curriculum:



#21 Writing Performance Tasks

The performance task of a Gold Seal Lessons includes an overview and a description.

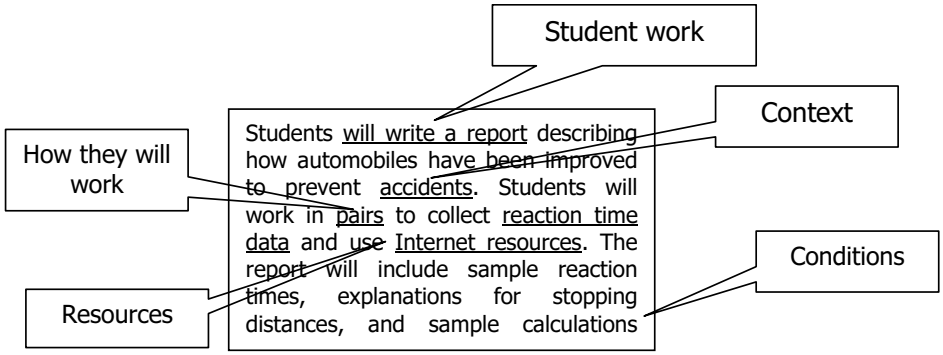
The overview is a description of how a student is expected to demonstrate learning (understanding, knowledge and skills). The task may be a product, performance or extended writing that requires rigorous thinking and relevance application. It is usually written in the third person describing the learning to other educators.

The **overview** includes:

- Student work that will be produced or performed
- Specific learning context
- Whether group or individual (how they will work)
- Resources students will be provided or have to acquire
- Setting where students will complete the work
- Conditions (often real-world) under which the work will be done

The **overview** does not include:

- Assessment, it is implied but not specified
- Specific direction to the student
- Specific equipment list
- Homework or reading assignments



The **description** of the performance task is the teacher procedures. The procedures can be step-by-step instructions on how to implement the lesson. Embedded in the procedures are instructional strategies and literacy strategies.

Sample

Description

Step One: Set the stage
Ask students if they have ever been in a car accident. Ask a few students to share with the class what happened. If any of them have been in a rear end collision, they are one in 2.5 million that occur each year. Although a rear end collision is the least fatal type of auto accident, they are the most common incidents on the road today. Explain to the students that they will be conducting an experiment related to human reaction time and how this affects automobile safety.

Step Two: Create groups and hold experiment
Students will work in teams of two. Students will use rulers to collect their reaction times. One person will hold a ruler from the top edge while the second student holds his/her thumb and index finger open at the bottom of the ruler. The second student will catch the ruler between his/her fingers when the first student releases it. The distance on the ruler will be recorded, and using a given table of data, converted to a reaction time. Each student should make five drops, recording the distance and reaction time for each drop. Each student should calculate his/her average reaction time (*see attached activity sheet*).

Step Two: Analyze the data
Once the testing is complete. The teacher should ask each student to record their test data and discuss their data and describe what they see. A discussion about the conversion chart should follow. Suggested questions are:

What method(s) did you use to determine your reaction time?
What do you notice about the conversion chart data?
What factors influence the ruler drop?
How accurate is your data?

Step Three: Using formulas
To generate more accurate data, ask students to use the following formula:

$$y = \frac{1}{2}gt^2$$

In the formulas, t = time (in seconds); y = distance (in cm); g = 980 cm/sec^2 (acceleration due to gravity). [Note: you can also use inches in your distance measurement, but you must change g to equal 385.8 in/sec^2.]
This formula provides the distance an object will fall in a given amount of time.

By rearranging Formula 1, you can get the amount of **time** it takes an object to fall a certain distance. All you have to do is plug in the distance (in either centimeters or inches) that the ruler fell into Formula 2 - this will give you the reaction time.

$$t = \sqrt{\frac{2y}{g}}$$

Ask students to compare their reaction times using the conversion chart and the formulas.

Step Four: Real-World Scenario
Ask students to solve the following problem.
Suppose a person is driving a car at 55 mph (80.67 feet/sec) during the day on a dry, level road. He sees a pedestrian and applies the brakes. What is the shortest stopping distance than can reasonably be expected? Total stopping distance consists of three components:

- *Reaction Distance*. First. Suppose the reaction time is 1.5 seconds. This means that the car will travel 1.5 x80.67 or 120.9 feet before the brakes are even applied.
- *Brake Engagement Distance*. Most reaction time studies consider the response completed at the moment the foot touches the brake pedal. However, there is an additional time required for the pedal to depress and for the brakes to engage. This is about .3 second, adding another 24.2 feet.
- *Physical Force Distance*. Once the brakes engage, the stopping distance is determined by physical forces ($D=S^2/(30*f)$) as 134.4 feet.

Total Stopping Distance = 120.9 ft + 24.2 ft + 134.4 ft = 279.5 ft

Step Five: Research and writing the report
Based on the data and experiences that the students have gathered so far, ask them to write a report responding to the following two questions:

- How have automobiles been improved to help prevent accidents, like rear end collisions from occurring?
- Can you think of a new car design that could help prevent accidents that result from slow human reaction time?

Activity:
Identify the Components of the Performance Task Overview

Underline evidence for each component, then draw a line to the corresponding component.

Example One: Exponential Growth and Decay

Student work | Context

Students will present to the class predictions and solutions on local, state or national issues that involves exponential decay or growth. The student must research the issue, collect data and create a model to represent this change.

How they will work | Resources | Conditions

Example Two: Healthy Snack Machines

Student work | Context

Students will design a poster of a circle graph on the topic of "Healthy Snacks in Snack Machines" based on a survey of at least 100 students regarding which snacks they prefer. Make recommendations to the principal about which snacks should be put into school machines, using data and graph.

How they will work | Resources | Conditions

GOLD SEAL LESSON

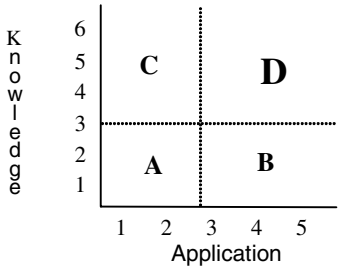
Algebra and the Music Industry

Subject(s)

Mathematics

Grade Level 6–7

Rigor/Relevance Framework



Instructional Focus

Number Operation and Concepts: Students use number, number sense, and number relationships in a problem-solving situation. Students communicate the reasoning used in solving these problems.
Algebraic Concepts and Relationships: Students use algebraic methods to investigate, model, and interpret patterns and functions involving numbers, shapes, data, and graphs in a problem-solving situation. Students evaluate and communicate the reasoning used in solving these problems.
Writing: Students write for a variety of purposes and audiences with sophistication and complexity appropriate to the grade level.

Student Learning

- The student will write equations and formulas that represent real-world situations.
- The student will evaluate formulas for given values.
- The student will choose appropriate measures of central tendency to describe a data set.
- The student will use mathematical evidence to support a point of view.
- The student will become aware of the ways that mathematics is used in real-world industries.

Performance Task

Overview
Students will read about the music business and will learn about the mathematical formulas used to calculate how much money is made by performing artists and record companies. Working with the teacher and their fellow classmates, students will write and solve equations based on these formulas.

Description
After students are familiar with the formulas used in the music business, they are asked to draw conclusions about the amount of money that a recording artist can actually make from the sale of CDs. They will do this by writing a paper answering the essential question: "How easy is it for a recording artist to get rich from just selling albums?"